Journal of Financial Intermediation **10**, 85–104 (2001)
doi:10.1006/jfin.2000.0303, available online at http://www.idealibrary.com on IDEAL®

Are There Optimal Multiple-Reserve Requirements?[1]

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Received May 12, 1997

A number of developing countries have adopted deficit finance regimes involving multiple-(currency and bond) reserve requirements. A key characteristic of these regimes is that the real interest rates on reservable bonds are higher than the real return rates on currency, so that the nominal interest rates on the bonds are positive. We seek an efficiency-based explanation for the existence of multiple-reserve regimes and for this key characteristic. We find that there are economies in which some of the efficient allocations can be supported only by multiple-reserve requirements, and that positive nominal bond rates may be needed to support some of these allocations. We also find that there are economies in which allocations supported by multiple-reserve regimes with negative nominal bond rates Pareto dominate single-reserve allocations, even when the latter are efficient relative to other single-reserve allocations. *Journal of Economic Literature* Classification Numbers: E42, E58, H62. © 2001 Academic Press

1. INTRODUCTION

A multiple-reserve requirements regime is a monetary regime in which the banking system is required to hold reserves of two different types: government currency and government bonds. Multiple-reserve regimes have been adopted by a number of developing countries at various times in recent years: examples include

[1] We thank the managing editor and an anonymous referee for helpful comments. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System.

1042-9573/01 $35.00

Chile, Korea, Mexico, and Pakistan. In each case, the country had a large public sector deficit and was using seigniorage to finance a substantial portion of the deficit.

The government agencies that administer multiple-reserve requirements often have considerable discretion about the levels of the two reserve ratios and/or the interest rates on reservable bonds. One regularity of observed multiple-reserve regimes is that the real interest rates on reservable bonds are higher than the real return rate on government currency, so that the nominal interest rates on the bonds are positive. We will refer to regimes of this type as "conventional." While the fact that the nominal interest rates on *private* bonds must be positive makes conventionality seem natural, nothing about the structure of multiple-reserve regimes seems to force the government to pay positive nominal rates on reservable government bonds. Typically, the government designates a particular class of bonds that banks are required to hold as reserves. Since the government is free to offer different and higher-yielding bonds to nonbank lenders, it should also be free to pay the banks any nominal bond rate it chooses, either positive or negative. In practice, however, we do not see reservable bonds with negative nominal interest rates.

The principal purpose of this paper is to use welfare analysis to explain why governments choose multiple-reserve requirements, and why they always pay positive nominal interest on reservable bonds. Our analysis follows Freeman (1987), Mourmouras and Russell (1992), and Espinosa (1995) by assuming that the government uses reserve requirements to help finance a budget deficit through seigniorage, and that the government's monetary policy decisions are based explicitly on considerations of public welfare. Our analytical framework is a simple nonstochastic overlapping generations model of fiat money and privately intermediated credit. We use this framework to study two different types of reserve requirements regimes. In the simplest "single-reserve" regimes, the government controls the real rate of return on fiat currency and sets the level of a currency reserve requirement. In the alternative "multiple-reserve" regimes, the government has access to these two policy instruments, plus two others: the level of a bond reserve requirement and the nominal interest rate on reservable bonds.[2]

Our analysis is based on the assumption that the government chooses policy settings that are efficient, given the alternatives available to it. For our purposes, a policy setting is efficient if the equilibrium allocation it supports cannot be Pareto dominated by the allocation supported by any other policy setting from any available policy regime.[3] Except where noted, we will assume that the set of policy regimes available to the government includes both single- and multiple-reserve regimes.

[2] Our model is essentially identical to the model developed by Espinosa (1995). Espinosa's model is similar to a model devised by Freeman (1987). The key features that distinguish Espinosa's model from Freeman's are the presence of heterogeneity within generations and the possibility that the central bank might impose bond reserve requirements.

[3] The government is assumed to face a fixed deficit that it must finance through seigniorage. The deficit is assumed to be too large to be financed without reserve requirements.

Since there will usually be more than one efficient allocation, the assumption that the government selects an efficient allocation raises the question of how it might choose from among the alternatives available. We assume that each government ranks efficient allocations based on their implications for the welfare of different socioeconomic groups. If governments behave in this way, then we should expect to see multiple-reserve requirements only in economies where they are the only reserve requirements regime that supports some of the efficient allocations.[4] We also consider the possibility that the government chooses single-reserve policy settings in the manner just described, but faces political constraints that prevent it from changing regimes (that is, from switching to multiple reserves) unless it can promise to increase the welfare of every socioeconomic group. If this is this case, then we might expect to see multiple-reserve requirements only in a subset of the economies just described—the subset in which there are multiple-reserve allocations that can Pareto improve efficient single-reserve allocations.

In light of these possibilities, the basic goal of our paper is to determine whether the model we study has specifications in which some of the efficient allocations can be supported only by multiple-reserve requirements. The hypothesis guiding our search for these specifications is that the increased policy-setting flexibility the government gains by adopting multiple-reserve requirements—the power to control the return rate on a second reservable asset, and to control the fraction of the reserves that consist of that asset—may give it a broader range of options for earning seigniorage revenue at the expense of different groups of agents. Given the prevalence of conventional multiple-reserve regimes, we will be especially interested in finding specifications in which some of the efficient allocations can be supported only by multiple-reserve policy settings featuring positive nominal interest rates on reservable bonds. The hypothesis guiding this search is that the government can use reservable bonds with relatively high real return rates to conduct more effective price discrimination in favor of the depositors of the banks that hold these bonds.[5]

As we have indicated, however, the fact that we do not observe unconventional multiple-reserve requirements does not make them inconceivable. For this reason, we will also look for specifications in which some of the efficient allocations can be supported only by unconventional multiple-reserve regimes. If our model rules out such specifications, then they may provide a relatively complete explanation for the characteristics of observed reserve requirements regimes. If it does not rule them out, then its implications may help policymakers identify situations in which they can improve social welfare by structuring multiple-reserve regimes in ways they may not have considered previously.

[4] For a given economy, the identity of the highest-ranking efficient allocation(s) will be different for different governments—that is, for different ranking schemes—and there will always be some scheme with standard properties under which a particular efficient allocation is optimal.

[5] Bryant and Wallace (1984) use a price discrimination argument of a somewhat different sort to provide an explanation for the minimum denominations on government bonds.

The next section of this paper presents an abbreviated description of a basic reserve requirements model. Section 3 presents our analysis of the model, and Section 4 offers some concluding remarks. The proofs of the paper's two propositions are presented in the Appendix, as are three examples.

2. MODEL

We analyze a two-period overlapping generations model with limited intragenerational heterogeneity and a number of legal/technological constraints on intertemporal trades. The model is essentially identical to that described by Espinosa (1995). Economic activity occurs at discrete dates $t = 1, 2, \dots$. At each date t a generation of agents is born; these "members of generation t" live during dates t and $t + 1$. Each generation of agents consists of N_{p} "poor savers" and N_{r} "rich savers." Rich savers differ from poor savers only in the magnitude and time distribution of their endowments of the single consumption good. The endowment patterns of rich and poor savers are invariant to the dates at which these agents are born. At each date an arbitrary number of private, competitive banks are operating in the economy. These banks may hold one or more of the following types of assets: private one-period bonds, which are available on the international credit market at an exogenously determined gross real interest rate $R > 1$,[6] government currency, which yields a gross real return rate $R_{\mathrm{m}}(t) > 0$ that is determined by the government through its ability to control the growth rate of the stock of currency, and government one-period bonds, which yield a gross real return rate $R_{\mathrm{b}}(t) > 0$ that is specified by the government.[7]

The liabilities of the banks consist of deposits that are offered to the public at a competitively determined gross real interest rate $R_{\mathrm{d}}(t)$. The banks are assumed to have zero operating costs and to maximize their date t profits, which must be zero in equilibrium.

The government is assumed to have imposed a legal minimum denomination on the real market value of a bank deposit. The endowments of the poor savers are assumed to be too small to permit them to purchase bank deposits. It is further assumed to be illegal and/or infeasible for them to pool their funds to purchase deposits or to finance deposit purchases with unsecured credit.[8] Rich savers' endowments are assumed to be large enough that this minimum denomination is irrelevant to them. Private and government bonds are assumed to have larger minimum denominations that make them inaccessible to any agents except banks. Thus

[6] Note that $R > 1$ implies that the net rate of return in the international credit market exceeds the net rate of growth of the economy, which, under these assumptions, is zero.

[7] Formally, these are nominal bonds, but since the central bank sets their nominal yield and has perfect foresight regarding the currency inflation rate, it effectively sets the real interest rate on the bonds (see below).

[8] For earlier examples of the use of minimum denomination restrictions of this type to generate demand for government currency, see Sargent and Wallace (1982) and Bryant and Wallace (1984).

the only asset available to poor savers is government currency, while rich savers may purchase government currency and/or bank deposits.

The aggregate real saving (first-period asset demand) functions of the poor and rich savers are denoted $m(R_{\mathrm{m}}(t))$ and $d(R_{\mathrm{k}}(t))$, respectively, where $R_{\mathrm{k}}(t) \equiv \max\{R_{\mathrm{m}}(t), R_{\mathrm{d}}(t)\}$. These functions are assumed to be continuous and strictly increasing for $R_{\mathrm{m}}(t) \geq \underline{R}_{\mathrm{m}}$ and $R_{\mathrm{k}}(t) \geq \underline{R}_{\mathrm{k}}$, with $0 \leq \underline{R}_{\mathrm{m}} < 1$ and $0 \leq \underline{R}_{\mathrm{k}} < R$. If $\underline{R}_{\mathrm{m}} > 0$ then we assume $m(R_{\mathrm{m}}(t)) = 0$ for $0 \leq R_{\mathrm{m}}(t) \leq \underline{R}_{\mathrm{m}}$ and similarly for $\underline{R}_{\mathrm{k}} > 0$ and $d(\cdot)$.[9]

The government is assumed to finance a fixed real deficit of $G > 0$ per period by issuing bonds and/or currency. The aggregate nominal stock of currency in circulation at date t is denoted $M(t)$; the date t price of a unit of the consumption good in terms of government currency (the date t price level) is denoted $p(t)$. Thus $R_{\mathrm{m}}(t) \equiv p(t)/p(t+1)$. The government is assumed to increase the stock of currency at a constant gross rate $z \geq 1$, so that $M(t) = zM(t-1)$ for all $t \geq 1$.

Government bonds are payable in government currency: a bond is a title to one unit of currency next period. The aggregate face value of the government bonds issued at date t is denoted $B(t)$. The currency price of a bond issued at date t, which is chosen by the government, is denoted $P_{\mathrm{b}}(t)$. The gross nominal interest rate on such bonds is $R_{\mathrm{nom}}(t) \equiv 1/P_{\mathrm{b}}(t)$; note that $R_{\mathrm{b}}(t) = R_{\mathrm{nom}}(t)R_{\mathrm{m}}(t)$. Government seigniorage revenue at dates $t \geq 2$ is given by

$$[M(t) - M(t-1)]/p(t) + [P_{\mathrm{b}}(t)B(t) - B(t-1)]/p(t).$$

The welfare of the poor and rich members of any generation t is assumed to be strictly increasing in $R_{\mathrm{m}}(t)$ and $R_{\mathrm{k}}(t)$, respectively. It is assumed that at date 1 there are an arbitrary number of "initial old" agents (the members of "generation 0") who live for one period and are endowed, in aggregate, with an exogenous initial stock of government currency $M(0)$. The welfare of these agents is assumed to be strictly increasing in $1/p(1)$, the inverse of the initial price level, which determines the purchasing power of the initial stock of currency.

The government is assumed to impose bond and/or currency reserve requirements on private banks. The fractions of a bank's assets that it is required to hold in the form of currency and government bonds are denoted θ_{m} and θ_{b}, respectively. We assume $\theta_{\mathrm{m}}, \theta_{\mathrm{b}} \in [0, 1]$ and $\theta \equiv \theta_{\mathrm{m}} + \theta_{\mathrm{b}} \in (0, 1)$. Each reserve ratio is the minimum ratio of the market value of a bank's holdings of one of the reservable liabilities (currency or bonds) to the market value of its entire portfolio of liabilities.

[9] The purpose of these assumptions is to allow us to use examples involving asset-demand functions that have negative values at low positive gross return rates. We are implicitly assuming that agents may borrow on the international credit market, unintermediated, at gross rate R. For example, if $R_{\mathrm{m}} \leq \underline{R}_{\mathrm{m}}$ then poor savers may choose to consume their endowments or to borrow on the international credit market at R; in either case their real currency balances will be zero. Similarly, if $R_k \leq \underline{R}_k$ then rich savers may choose to consume their endowments or borrow at R; in either case, the assets of domestic intermediaries will be zero and their real reserves of fiat currency will also be zero.

We will confine ourselves to the study of stationary equilibria with reserve requirements; hereafter, simply "reserve requirements equilibria." These are competitive equilibria in which (1) the rate of return on private bonds exceeds the rates of return on government currency or bonds, so that the banks will hold government liabilities only to meet the reserve requirements, (2) the rate of return on bank deposits exceeds the rate of return on government currency, so that rich savers will hold only bank deposits, and (3) the values of all real variables and all nominal return rates are constant, while the values of all nominal variables except return rates grow at a common fixed rate (which may be zero).[10] Given R, G, and $M(0)$, a reserve requirements equilibrium can be characterized as values of the government policy variables z, P_{b}, θ_{m}, and θ_{b} that satisfy $z \in [1, \infty)$, $P_{\mathrm{b}} \in (0, \infty)$, $\theta_{\mathrm{m}}, \theta_{\mathrm{b}} \in [0, 1]$, and $\theta \equiv \theta_{\mathrm{m}} + \theta_{\mathrm{b}} \in (0, 1)$, plus values of the endogenous variables R_{m}, R_{b}, R_{d}, and $m_0 \equiv M(0)/p(1)$ that satisfy $m_0 \geq 0$,

$$R_{\mathrm{m}} = \frac{1}{z}, \tag{1}$$

$$R_{\mathrm{b}} = \frac{R_{\mathrm{m}}}{P_{\mathrm{b}}}, \tag{2}$$

$$R_{\mathrm{b}} < R, \tag{3}$$

$$R_{\mathrm{d}} = (1 - \theta_{\mathrm{m}} - \theta_{\mathrm{b}})R + \theta_{\mathrm{b}} R_{\mathrm{b}} + \theta_{\mathrm{m}} R_{\mathrm{m}}, \tag{4}$$

$$R_{\mathrm{m}} < R_{\mathrm{d}} < R, \tag{5}$$

$$G = (1 - R_{\mathrm{m}})\,[m(R_{\mathrm{m}}) + \theta_{\mathrm{m}} d(R_{\mathrm{d}})] + (1 - R_{\mathrm{b}})\theta_{\mathrm{b}} d(R_{\mathrm{d}}), \tag{6}$$

and

$$m_0 = m(R_{\mathrm{m}}) + (\theta_{\mathrm{m}} + \theta_{\mathrm{b}}) d(R_{\mathrm{d}}) - G. \tag{7}$$

The first equation follows from the fact that in a steady-state equilibrium, aggregate real balances $M(t)/p(t)$ must be constant. The second equation rules out profitable arbitrage in the government currency and bond markets. Inequalities (3) and (5) guarantee that banks hold government liabilities only as legal reserves and that rich savers hold only bank deposits. The fourth equation expresses the relationship between the interest rate on bank deposits, the two reserve ratios, and the rates of return on the three nonbank assets that is implied by the assumption that banks earn zero profits. The sixth and seventh equations ensure that the government meets its budget constraint at dates $t \geq 2$ and $t = 1$, respectively.

In a reserve requirements equilibrium we have $p(t)/p(t+1) = R_{\mathrm{m}} = 1/z$, $R_{\mathrm{nom}}(t) = R_{\mathrm{nom}} \equiv R_{\mathrm{b}}/R_{\mathrm{m}}$, $M(t)/p(t) = m(R_{\mathrm{m}}) + \theta_{\mathrm{m}} d(R_{\mathrm{d}})$, and $B(t)/p(t) = \theta_{\mathrm{m}} d(R_{\mathrm{d}})$ for all $t \geq 1$. These equations imply that $M(t+1)/M(t) = B(t+1)/$

[10] It is easy to generalize the model to cover situations in which the values of real variables grow at fixed, exogenously determined rates.

$B(t) = 1/R_m = z$ for all $t \geq 2$. Note that under our assumptions, the government can use its control over z and P_b to set R_m and R_b at any values between 0 and R. For purposes of simplicity, we will think of R_m and R_b as policy variables.

Note that any positive reserve requirement is binding on the rich savers, since the banks will not hold government liabilities in the absence of such a requirement. A stationary equilibrium without reserve requirements would consist of values of R_m and $p(1)$ such that $G = (1 - R_m)m(R_m)$ and $m_0 = m(R_m) - G$; the government would have to earn all its seigniorage revenue from the poor savers. For simplicity, we assume that the value of G is large enough to rule out equilibria of this type.

Hereafter, we will describe a "reserve requirements policy setting" as values $(R_m, R_b, \theta_m, \theta_b)$ that satisfy $R_m \geq 0$, $R_b \geq 0$, $\theta_m, \theta_b \in [0, 1]$, and $\theta \equiv \theta_m + \theta_b \in (0, 1)$, and a "reserve requirements allocation" as the values (G, R_m, R_d, m_0) that can be supported as a reserve requirements equilibrium. A reserve requirements policy setting is a "single-reserve policy setting" iff $\theta_b = 0$ and/or $R_b = R_m$; it is a "multiple-reserve policy setting" iff $R_b \neq R_m$ and $\theta_b > 0$.[11] A single-reserve policy setting can be characterized by values (R_m, θ). A reserve requirements equilibrium supported by a single- (multiple-) reserve policy setting is a "single- (multiple-) reserve equilibrium," and a reserve requirements allocation supported a single- (multiple-) reserve equilibrium is a "single- (multiple-) reserve allocation." We will assume that the value of G is small enough to ensure that each specification has at least one equilibrium of each type.

We will refer to a reserve requirements allocation as "efficient" if it is Pareto optimal relative to all other reserve requirements allocations. In some cases, we may refer to a single-reserve allocation as "single-reserve efficient" if it is Pareto optimal relative to all other single-reserve allocations. As we shall see, an allocation can be single-reserve efficient without being efficient by the broader definition.

3. ANALYSIS

Before beginning the analysis, it may be useful to review the monetary policy problem confronting the government. Monetary policy must accommodate fiscal policy, so it must be conducted in a way that allows the government to earn a fixed amount of real revenue G from seigniorage. The government is also concerned about the welfare of the three groups of agents: the poor savers, the rich savers, and the initial old. The welfare of the poor savers depends entirely on the real rate of return on currency R_m. The welfare of the initial old depends on the real demand for bonds and/or currency, which depends largely, though not entirely, on the level of the aggregate reserve ratio θ. The welfare of the rich savers depends

[11] Note that according to our definition, a policy setting in which $R_b \neq R_m$ and $\theta_m = 0$ is a multiple-reserve policy setting, even though there will be only one asset (bonds) held as reserves. For an analysis of these "single-bond reserve requirements," see Espinosa and Russell (1999).

entirely on the real deposit rate R_{d}, which is influenced by both R_{m} and θ. In a single-reserve regime, these are the only policy variables that influence the real deposit rate. In a multiple-reserve regime, however, the real deposit rate is also influenced by two additional policy variables. One of these is the real reservable bond rate R_{b}; the other is $\theta_{\mathrm{b}}/\theta$, the fraction of a bank's total reserves that must consist of bond reserves.

As we indicated in the Introduction, the goal of our analysis is to search for specifications of the model in which some of the efficient allocations can be supported only by multiple-reserve policy settings. In economies like these, it may be possible for an optimizing government to increase social welfare by adopting multiple-reserve requirements instead of single reserve requirements.

Why might we expect to be able to find specifications of this type? As we have just noted, under either type of reserve requirements regime the welfare of the poor savers is determined exclusively by the rate of return on currency, while the welfare of the initial old agents is profoundly influenced by the aggregate reserve ratio. Under a single-reserve regime, there is no way the government can change the deposit interest rate, which determines the welfare of the rich savers, without changing the values of these two variables. Conversely, there is no way it can change the value of either variable without changing the deposit interest rate. Under a multiple-reserve regime, however, it is possible for the government to use changes in the bond reserve fraction and/or the reservable bond rate to change the deposit rate without changing the currency return rate or the aggregate reserve ratio, and it is also possible for the government to change either of these variables without changing the deposit rate. It seems reasonable to suspect that increasing the government's ability to manipulate these three welfare-determining variables independently might allow it to support efficient allocations that would not be available otherwise.

For reasons described in the Introduction, we are looking with particular interest for specifications that have efficient allocations that can be supported only by conventional multiple-reserve requirements. To see why we might expect to find them, imagine that the government delegates its monetary policy decisions to a monetary authority that behaves in the manner described in the Introduction: it chooses the efficient allocation that ranks highest on its social welfare scale. Up to now, however, the government has not given the monetary authority the power to impose bond reserve requirements. Consequently, the current reserve requirements policy setting is a single-reserve setting that is efficient, relative to other single-reserve allocations, and ranks higher than any of these allocations. The monetary authority could engineer higher-ranking allocations if it could change policy in a way that increased the welfare of the initial old at the expense of the poor savers, without hurting the rich savers, but the limitations of single-reserve regimes prevent it from doing this. To increase the welfare of the initial old, the government must increase the reserve ratio. A higher reserve ratio reduces the deposit rate, which hurts the rich savers, but it also produces more revenue from seigniorage. The government needs to find a way to use this increased revenue to finance a second policy change that restores the deposit rate to its original level. Under a single-reserve regime, there is only one other policy instrument available: the currency return rate, which will have to be increased. Unfortunately for the government, an increase in the currency return rate also reduces the amount of seigniorage revenue from the poor savers, whose contribution to total seigniorage revenue is large and whom the government is not interested in helping. Any increase in the currency return rate large enough to restore the deposit rate to its original level will result in a substantial net loss of seigniorage revenue, leaving the government unable to cover its deficit.

Now suppose that the government gives the monetary authority the power to impose multiple-reserve requirements. It responds by increasing the aggregate reserve ratio, as above, but it also converts part of the new aggregate reserve ratio into a bond reserve ratio. The government can now increase the deposit rate by increasing the real interest rate on reservable bonds instead of increasing the real currency return rate. As a result, it can return the deposit rate to its original level without losing any seigniorage revenue from the poor savers. There will be a net loss of revenue from the rich savers, but this loss can be offset by reducing the currency return rate, which will increase the revenue from the poor savers.[12] So the government has accomplished its goal. It has increased the welfare of the initial old, who benefit from the higher aggregate reserve ratio, at the expense of the poor savers, who are hurt by the lower currency return rate. It has done this without hurting the rich savers, who face an unchanged deposit rate, and since the real reservable bond rate is higher than the real currency return rate, the new multiple-reserve regime is conventional in nature.

A situation of this type is described in Example 1. In this example, the government monetary authority ranks efficient allocations using a social utility function with standard features.[13] Initially, the monetary authority is assumed to have access only to single-reserve requirements. Thus, the initial allocation is a single-reserve allocation that produces higher social utility than any other allocation of that type, but there are multiple-reserve allocations that produce higher levels of social utility. The utility-maximizing (and thus, efficient) multiple-reserve allocation features a higher aggregate reserve ratio and a lower initial price level. The currency return rate is also lower, but the deposit rate is virtually unchanged. This is possible

[12] The cause of the net loss of revenue from the rich savers is a phenomenon described by Freeman (1987). In models of this type, seigniorage is an inefficient tax because the government liabilities that provide its tax base are socially inefficient: their pretax of return is lower than the pretax rate of return on private liabilities. Increasing the aggregate reserve ratio deepens this inefficiency by forcing agents to substitute inefficient assets for efficient ones. As a result, setting the reservable bond rate at a level that exactly rolls back the increase in seigniorage revenue produced by the higher reserve ratio does not restore the deposit rate to its original level. Complete restoration of the deposit rate requires an even higher reservable bond rate. The higher reservable bond rate must be financed by a lower currency return rate that increases the seigniorage revenue from poor savers.

[13] The class of social utility functions we use in our examples is described at the beginning of the second section of the Appendix.

because the government has converted part of the currency reserve requirement into a bond reserve requirement, and it has issued reservable bonds with positive nominal interest rates. If the government had tried to restore the deposit rate to its original level by increasing the real currency return rate, then it would have suffered a large loss of revenue.

Although examples like this one can help explain the existence of conventional multiple-reserve regimes, they do not help explain our failure to observe unconventional regimes. As our second example demonstrates, the economies we study may also have efficient allocations that can be supported only by multiple-reserve regimes in which the reservable bonds have negative nominal rates.

Example 2 also illustrates a different way in which the government may be able to use multiple-reserve requirements to achieve welfare improvements. In the economy of this example, the social-utility-maximizing single-reserve regime involves a relatively low real rate of return on currency, and thus a relatively high inflation tax rate. In fact, the inflation tax rate is so high that a marginal increase in the real currency return rate would cause the seigniorage revenue from poor savers to rise.[14] Under a single-reserve regime, however, increasing R_{m} would cause *total* seigniorage revenue to fall: the loss of seigniorage revenue from rich savers would exceed the gain from the poor savers. The government could recover the lost revenue by increasing the reserve ratio, but the required increase would be large enough to cause the deposit rate to fall. Thus, there is no way for the government to help the poor savers without hurting the rich savers, and the terms of this welfare tradeoff are such that any attempt to exploit it would cause social utility to fall.

Under a multiple-reserve regime, in contrast, the government can increase the return rate facing poor savers without increasing the return rate facing rich savers or the seigniorage revenue earned from them. First, it must convert part of its currency reserve requirement into a bond reserve requirement, and some of its outstanding currency into bonds. Of course, this conversion will not matter if both liabilities have the same real return rate, but the government is now free to increase the real currency return rate without changing the average tax rate on rich savers, because increases in R_{m} can be offset by decreases in the real bond rate R_{b}. The increases in R_{m} increase poor savers' currency demand, which increases the welfare of the initial old agents. In addition, the increased seigniorage revenue from poor savers allows the government to give up some revenue from rich savers, which means that it can set the reservable bond rate at a level that produces a higher deposit rate. Thus, in this example, increasing the currency return rate independently from the real bond rate can produce multiple-reserve allocations that Pareto dominate the initial single-reserve allocation. However, the multiple-reserve policy settings that

[14] An increase in the real currency return rate reduces the inflation tax rate on poor savers, but it also increases base for the tax by causing poor savers' demand for currency to rise. In this example, at the initial real currency return rate the tax base effect is stronger than the tax rate effect.

support these allocations are unconventional in nature: the nominal interest rates on reservable bonds are negative.[15]

Examples 1 and 2 establish that there are indeed situations in which some of the efficient allocations can be supported only by multiple-reserve requirements. Example 1 establishes that in some of these situations, the supporting multiple-reserve policy settings can feature positive nominal interest rates on reservable bonds. Example 2 establishes that in other situations, the supporting policy settings can feature negative nominal bond rates.

The multiple-reserve allocations described in these examples can be supported by a continuum of different policy settings, each of which features different currency and bond reserve ratios and a different nominal bond rate. These examples are not special: indeterminacy of the supporting policy settings is a general characteristic of the multiple-reserve allocations in this model. Given our interest in allocations that can be supported by policy settings with positive nominal bond rates, this indeterminacy raises some important questions. Is it possible that some multiple-reserve allocations, or even all such allocations, can be supported by alternative policy settings featuring nominal bond rates with opposite signs? If so, is it possible that the government can always find a positive-nominal-bond-rate setting that will support a given efficient allocation? A result like this might provide a very convincing explanation for our failure to observe negative nominal bond rates. Conversely, is it possible that positive nominal bond rates are never necessary to support a given efficient allocation?

It turns out that the answer to all these questions is "no." As we indicate in the Appendix, every policy setting that supports the Example 1 allocation involves

[15] An interesting feature of Example 2 is that the ability of independent increases in the real currency return rate to produce Pareto improvements does not stop at the point where R_m is no longer set at a value from which an increase would cause seigniorage revenue from poor savers to rise. Thus, avoiding single-reserve policy settings in which R_m is set at such values is necessary, but not sufficient, for the government to achieve an efficient reserve requirements allocation. For R_m values somewhat higher than the value that maximizes the revenue from the poor savers, a further increase in R_m produces a relatively small decline in revenue but a relatively large increase in poor savers' currency demand. The lost revenue can be recovered from the rich savers, without reducing their welfare, by reducing both the reservable bond rate R_b and the aggregate reserve ratio θ in a way that holds the deposit rate constant. Although this second change in policy causes rich savers' reserve demand to decline, the decline is more than offset by the increase in currency demand from the poor savers. This situation reflects the phenomenon described by Freeman (1987) that was discussed in Footnote 13 above. In a single-reserve regime, a simultaneous reduction in R_m and θ that holds R_d constant will always increase the amount of seigniorage revenue, while a reduction in both policy variables that holds seigniorage revenue constant will always increase the deposit rate. It follows that if the government is not concerned about the level of money demand (that is, if it does not care about the welfare initial old) then it should always reduce both variables as far as it can, which turns out to be a point where θ is positive but R_m is zero. In Example 2, the reserve requirements allocation that maximizes social utility can be produced by increasing R_m, and decreasing R_b and θ, in the manner just described. The currency return rate is now high enough to eliminate free lunches. Given the government's budget constraint, there is no way to increase R_m further without reducing the total demand for government liabilities, which hurts the initial old.

positive nominal bond rates, and every policy setting that supports the Example 2 allocation involves negative nominal rates. More generally, we can show that every policy setting that supports a given multiple-reserve allocation features a nominal bond rate with the same sign: they are either all positive or all negative. It follows that we can divide multiple-reserve allocations into two categories: allocations supportable only by conventional multiple reserves regimes, and allocations supportable only by unconventional regimes.

We establish this result formally as our Proposition 1:

PROPOSITION 1. *Suppose a reserve requirements allocation* $(\bar{G}, \bar{R}_{\mathrm{m}}, \bar{R}_{\mathrm{d}}, \bar{m}_0)$ *can be supported by a multiple-reserve requirements policy setting with* $\bar{R}_{\mathrm{b}} > \bar{R}_{\mathrm{m}}$. *Then the set of alternative multiple-reserve requirements policy settings that can support this allocation can be indexed by* $\hat{R}_{\mathrm{b}} \in [R_{\mathrm{b}}^{\min}, R]$, *where* $R_{\mathrm{b}}^{\min} > \bar{R}_{\mathrm{m}}$. *Suppose a reserve requirements allocation can be supported by a multiple-reserve requirement policy setting with* $\bar{R}_{\mathrm{b}} < \bar{R}_{\mathrm{m}}$. *Then the set of alternative reserve requirements policy settings that can support this allocation can be indexed by* $\hat{R}_{\mathrm{b}} \in [0, \hat{R}_{\mathrm{b}}^{\max}]$, *where* $\hat{R}_b^{\max} < \bar{R}_{\mathrm{m}}$.

The logic behind Proposition 1 is simple. Any reserve requirements allocation in our model can be uniquely characterized by a currency return rate, a deposit rate, and an aggregate reserve ratio. The currency return rate and the deposit rate uniquely determine the welfare of the poor savers and the rich savers, respectively. The welfare of the initial old depends on the level of real demand for nominal assets. The currency return rate uniquely determines small savers' currency demand; given the deposit rate, the aggregate reserve ratio determines rich savers' aggregate real currency and bond demand.

Under a multiple-reserve regime, one can think of the government as using a two-stage process to implement a policy that supports the allocation it has selected. In the first stage, it sets the real currency reserve rate and the aggregate reserve ratio at the unique levels R_{m}^* and θ^*, respectively, that support the allocation. In the second stage, it uses its two additional policy instruments—the real reservable bond rate R_{b}, and the fraction of total reserves that consist of bonds, $\theta_{\mathrm{b}}/\theta^*$—to produce the single desired deposit rate R_{d}^*. Not surprisingly, there are many ways to do this. However, the desired deposit rate is either higher or lower than $\bar{R}_{\mathrm{d}} \equiv (1-\theta^*)R + \theta^* R_{\mathrm{m}}^*$, the deposit rate in a single-reserve regime where the government imposes the target aggregate reserve ratio θ^* and the target currency return rate R_{m}^*. If the desired deposit rate is lower (higher) than $\bar{R}_{\mathrm{d}}$ then the real rate of return on bonds must be lower (higher) than R_{m}^*, because $\theta_{\mathrm{b}} R_{\mathrm{b}} + (\theta^* - \theta_{\mathrm{b}}) R_{\mathrm{m}}^*$ must be higher (lower) than $\theta^* R_{\mathrm{m}}^*$. Thus, while the real and nominal interest rates on government bonds will not be unique, it will be true either that all of the possible nominal rates must be positive or that all of them must be negative.

It is readily seen that if the equation $R_{\mathrm{d}}^* = (1-\theta^*)R + \theta_{\mathrm{b}} R_{\mathrm{b}} + (\theta^* - \theta_{\mathrm{b}}) R_{\mathrm{m}}^*$ has legitimate solutions for θ_{b} and R_{b} that feature $R_{\mathrm{b}} < R_{\mathrm{m}}^*$, then it will also have legitimate solutions featuring $R_{\mathrm{b}} = 0$. Thus, the range of unconventional multiple-reserve policy settings described in the second part of Proposition 1 always includes settings involving $\hat{R}_b = 0$. Stated differently, Proposition 1 establishes that any allocation supportable by an unconventional multiple-reserve requirement can be supported by a multiple-reserve policy setting in which the gross real rate of return on reservable bonds is zero. As Espinosa (1995) points out, if the gross real return rate on a reservable government liability is zero then the associated reserve requirement is equivalent to a direct proportional tax on deposits.[16] It follows that, in our model, any allocation supportable by an unconventional multiple-reserve requirement can be supported by a combination of a single-reserve requirement and a direct deposit tax. This implication of Proposition 1 may help explain our failure to observe negative nominal interest rates in actual economies. Governments that find themselves in situations in which issuing negative-nominal-interest bonds would increase social welfare may consider bonds of this sort so peculiar that they resort to direct deposit taxation instead.[17]

In Example 2, we describe an efficient allocation with two distinctive characteristics. First, it is achievable only through multiple-reserve requirements with negative nominal bond rates. Second, it Pareto dominates a single-reserve allocation that is efficient, relative to the other single-reserve allocations—an allocation that the government might select if its policy choices were restricted to single-reserve requirements. In Example 1, we describe an efficient allocation that can be supported only by multiple-reserve requirements with positive nominal bond rates. However, this allocation does not Pareto dominate the optimal single-reserve allocation, and it can be shown that it does not Pareto dominate any allocation that is single-reserve efficient. In these circumstances, it seems natural to wonder whether there might also be specifications of our model in which allocations supported by conventional multiple-reserve requirements Pareto dominate allocations that are single-reserve efficient—specifications, that is, in which the government might be able to improve efficiency by adopting conventional multiple-reserve requirements.

It turns out that we will not be able to find specifications of the type we are looking for. As we establish in Proposition 2, any single-reserve allocation that is Pareto dominated by a conventional multiple-reserve allocation is also Pareto dominated by another single-reserve allocation. Thus, a single-reserve allocation that is efficient relative to all other single-reserve allocations is also efficient relative to all other reserve requirements allocations.

[16] It is important to distinguish a *deposit tax*, which is collected when funds are deposited, from a *deposit returns tax*, which is collected when funds are returned to depositors. In our model, a deposit tax is equivalent to a reserve requirement with a zero gross real return rate on the reservable government liability, but a deposit returns tax is not. A deposit returns tax will not raise any government revenue at date 1 and thus is not, strictly speaking, consistent with a steady-state equilibrium as we have defined it. However, such a tax will produce higher welfare than a deposit tax in a steady state that abstracts from the situation at date 1: see Freeman (1987). Mourmouras and Russell (1992) also study deposit returns taxes, while Espinosa (1995) studies deposit taxes. Our discussion follows Espinosa (1995).

[17] A potential problem with this explanation for the absence of unconventional multiple-reserve regimes is the fact that the power to impose or change the level of direct taxes on deposits (or anything else) typically belongs to the fiscal authorities rather than to the monetary authority.

PROPOSITION 2. *Suppose an allocation supported by a single-reserve requirement with policy setting* $(\overline{R}_{\mathrm{m}}, \overline{\theta}_{\mathrm{m}})$ *is Pareto dominated by an allocation supported by a multiple-reserve requirement whose policy setting features either* (1) $\hat{R}_{\mathrm{b}} > \hat{R}_{\mathrm{m}}$ *or* (2) $\hat{R}_{\mathrm{b}} < \hat{R}_{\mathrm{m}} = \overline{R}_{\mathrm{m}}$. *Then there is an alternative single-reserve policy setting that supports an allocation that Pareto dominates the original single-reserve allocation.*

The logic behind Proposition 2 is suggested by Example 2, which describes two closely related types of situations in which the government can Pareto improve single-reserve allocations by switching to a multiple-reserve regime and setting the real reservable bond rate at a lower level than the real currency return rate. In one of these situations, increasing the currency return rate increases the seigniorage revenue from poor savers. In the other situation, increasing R_{m} decreases poor-saver seigniorage—but the decrease is slight, and the increased currency return rate produces a large increase in their currency demand. In both cases, increasing R_{m} independently from R_{b} allows the government to take advantage of the situation without losing revenue from the rich savers, but since the real currency return rate ends up higher than the real bond rate, the Pareto-improving multiple-reserve regimes are unconventional.

Why isn't it possible to have cases of the opposite type, in which the rich savers are taxed too heavily? In these cases, increasing the real bond rate independently from the currency return rate would increase the seigniorage revenue from rich savers, allowing conventional multiple-reserve regimes to produce Pareto improvements. It turns out that cases of this type *are* possible. When they occur, however, there is always another strategy for producing Pareto improvements that does not require changing reserve requirements regimes. This alternative strategy involves reducing the reserve ratio—another way to increase the welfare of the rich savers by reducing their implicit tax rate. It can be shown that whenever increasing the real bond rate will increase the seigniorage revenue from rich savers, reducing the reserve ratio will have the same effect. Since changes in the reserve ratio are irrelevant to the poor savers, a reduction in the ratio will have no effect on their welfare or on the amount of seigniorage revenue obtained from them. In addition, a reduction in the reserve ratio that increases the seigniorage revenue from rich savers must also increase their money demand, which benefits the initial old. Thus, when the rich savers are taxed too heavily, the government can solve the problem, and improve efficiency, without switching reserve requirements regimes. Example 3 describes a situation of this type.[18]

[18] At first glance, our Proposition 2 seems hard to reconcile with results obtained by Espinosa (1995). Espinosa derives a condition under which conventional multiple-reserve allocations can Pareto dominate single-reserve allocations, and he presents a class of examples. It turns out, however, that any single-reserve allocation that satisfies Espinosa's conditions is inefficient relative to other single-reserve allocations. Each of the single-reserve allocations from his class of examples is Pareto dominated by an alternative single-reserve allocation that features a lower reserve ratio. This alternative allocation is Pareto noncomparable to the multiple-reserve allocation that Pareto dominated the original single-reserve allocation. Our Example 3 is based directly on one of Espinosa's examples.

4. CONCLUSION

We conclude by summarizing the relationship between our results, the results reported in Freeman's (1987) analysis of single-reserve requirements, and the results reported in Espinosa's (1995) analysis of multiple-reserve requirements.

Freeman's analysis differs from Espinosa (1995), and from our analysis, in two important ways. First, Freeman's model lacks intragenerational heterogeneity, so an optimal seigniorage policy requires only one type of tax on saving. There is no role for a second (bond) reserve requirement, which derives its usefulness, in our model, from the fact that it allows the government to tax saving by one group of agents at a rate that is independent from the rate at which it taxes saving by the other group. As we have seen, Espinosa (1995) introduces intragenerational heterogeneity in order to provide a role for multiple-reserve requirements.

Second, Freeman implicitly assumes that policymakers give no weight to the effects of different policies on the welfare of the agents who are alive at the time the policies are implemented (the initial old). This assumption is critical to his distinctive findings, which are that the optimal reserve ratio is the smallest feasible ratio and that the optimal gross real currency return rate is zero (a hyperinflation). In Freeman's model, reducing the (single-) reserve ratio reduces the demand for fiat currency. Since the initial old are endowed with fiat currency, and monetary policy has no other effect on them, a policy that minimizes the reserve ratio also minimizes their consumption. Thus, a government that was sufficiently concerned about the welfare of the initial old might choose a single-reserve regime with a reserve ratio higher than the minimum feasible ratio, producing a positive gross real currency return rate.

Espinosa (1995) shows that his model produces a form of Freeman's result: if the government does not care about the initial old, then it is optimal for it to set the real return rate on reservable bonds at 0.[19] He also shows, using an argument similar to Freeman's, that increasing the gross real reservable bond rate increases the consumption of the initial old, other things equal. This result suggests that if the government cares about the initial old, then reservable bonds with positive gross real return rates might be part of an optimal policy. However, Espinosa does not establish that positive-gross-real-rate bonds are ever necessary for optimality.

Our analysis, which is conducted using Espinosa's model, extends these results in several ways. First, we show that there are efficient allocations that can be supported only by multiple-reserve requirements. This result verifies that multiple-reserve requirements can be necessary for optimality and thus provides a *prima facie* explanation for their existence. We also show that some of these allocations can be supported only by conventional multiple-reserve requirements, while others can be supported only by unconventional reserve requirements. Thus, there are situations in which the optimal reserve requirements policy necessarily involves

[19] Espinosa and Russell (1999) show that the analogy between the two models can be extended a bit further: it is also optimal for the government to set the currency reserve ratio at 0.

bonds with positive nominal interest rates—the only type of reservable bonds we actually observe. On the other hand, there are also situations in which negative nominal bond rates are necessary for optimality. However, we show that in situations of the latter type, the range of supporting multiple-reserve policy settings always includes settings in which the gross real rate of return on reservable bonds is zero. This result extends the Freeman/Espinosa results about the optimality of government liabilities with zero gross return rates to many situations in which the government cares about the welfare of the initial old. It may also have a more practical implication. A multiple-reserve requirement with a zero gross real bond rate is equivalent to a single-currency reserve requirement combined with a direct tax on bank deposits. It is possible that the reason we do not observe unconventional multiple-reserve requirements is that in economies where regimes of this sort would be optimal, the government levies deposit taxes instead.

Espinosa (1995) also shows that there are circumstances in which conventional multiple-reserve allocations can Pareto improve single-reserve allocations. We qualify this result and we also extend it. We begin by showing that any single-reserve allocation that can be Pareto improved by a conventional multiple-reserve allocation can also be Pareto improved by another single-reserve allocation. Thus, the finding that conventional multiple-reserve requirements can produce Pareto improvements is not robust to the assumption that the government chooses single-reserve policy settings in an efficient way. Stated differently, we show that this model cannot provide an explanation for conventional multiple-reserve requirements that is based solely on efficiency comparisons. We go on to provide an example in which an unconventional multiple-reserve allocation can Pareto improve an allocation that is single-reserve efficient. These results, combined with the results summarized in the second half of the preceding paragraph, suggest an interesting question for future empirical research on the properties of reserve requirements regimes: Is it common for governments to combine single-currency reserve requirements with direct taxation of deposits?

Our model is grounded on the assumption that government decisions about reserve requirements policy are based largely on considerations of seigniorage revenue, rather than financial stability or monetary control. For developing countries in which seigniorage provides a substantial fraction of government revenue, this assumption seems very reasonable. Goodfriend and Hargraves (1987) conclude that seigniorage considerations have also been a powerful force driving reserve requirements policy in the United States, even though the contribution of seigniorage to total U.S. government revenue has been relatively small. This conclusion raises the question of why we have not seen bond reserve requirements in the United States, and why the level of currency reserve requirements has gradually declined.

One answer to this question that seems plausible to us is that the United States has been in a uniquely favorable position to earn seigniorage without imposing reserve requirements. During the postwar period, the average real interest rate on U.S. government debt has been substantially lower than the average U.S. output growth rate, despite a debt/GDP ratio that has averaged almost 50%.[20] So the U.S. government has been able to earn substantial quantities of bond seigniorage without forcing U.S. banks to hold bond reserves. In addition, the nonbank public in the United States has been willing to hold substantial quantities of government currency, and this domestic demand has been augmented by very large foreign demand for U.S. currency.[21] Indeed, the period of relatively rapid decline in U.S. reserve requirements has roughly coincided with a period of growing foreign demand for U.S. currency. This situation may have allowed the U.S. government to substitute seigniorage earned from foreigners for seigniorage earned from U.S. banks and their domestic depositors.

APPENDIX

Proofs

Recall that $\theta \equiv \theta_{\mathrm{m}} + \theta_{\mathrm{b}}$. In what follows it is useful to define $A \equiv m(R_{\mathrm{m}}) + \theta d(R_{\mathrm{d}})$, which represents aggregate real balances of government liabilities, and to note that Eqs. (1)–(3) imply that $M(0)/p(1) = A - G = R_{\mathrm{m}}[m(R_{\mathrm{m}}) + \theta_{\mathrm{m}} d(R_{\mathrm{d}})] + R_{\mathrm{b}}\theta_{\mathrm{b}} d(R_{\mathrm{d}})$.

Proof of Proposition 1. The deposit interest rate equation $R_{\mathrm{d}} = (1-\theta)R + \theta_{\mathrm{m}} R_{\mathrm{m}} + \theta_{\mathrm{b}} R_{\mathrm{b}} +$ implies that, in any binding stationary multiple-reserve equilibrium, the government budget constraint can be rewritten $G = (1 - R_{\mathrm{m}})m(R_{\mathrm{m}}) + (1 - R)\theta d(R_{\mathrm{d}}) + (R - R_{\mathrm{d}})d(R_{\mathrm{d}})$. In the original single-reserve equilibrium we have $\bar{R}_{\mathrm{d}} = \bar{\theta}\bar{R}_{\mathrm{m}} + (1 - \bar{\theta})R$ and $M(0)/\bar{p}(1) = \bar{A} - \bar{G}$, where $\bar{A} = m(\bar{R}_{\mathrm{m}}) + \bar{\theta} d(\bar{R}_{\mathrm{d}})$ and $\bar{G} = (1 - \bar{R}_{\mathrm{m}})m(\bar{R}_{\mathrm{m}}) + (1 - R)\bar{\theta} d(\bar{R}_{\mathrm{d}}) + (R - \bar{R}_{\mathrm{d}})d(\bar{R}_{\mathrm{d}})$. A Pareto dominant multiple-reserve equilibrium would be $\hat{R}_{\mathrm{d}} = (1 - \hat{\theta}_{\mathrm{m}} - \hat{\theta}_{\mathrm{b}})R + \hat{\theta}_{\mathrm{m}}\hat{R}_{\mathrm{m}} + \hat{\theta}_{\mathrm{b}}\hat{R}_{\mathrm{b}} +$ and $M(0)/\hat{p}(1) = \hat{A} - \hat{G}$, where $\hat{A} = m(\hat{R}_{\mathrm{m}}) + \hat{\theta} d(\hat{R}_{\mathrm{d}})$ and $\hat{G} = (1 - \hat{R}_{\mathrm{m}})m(\hat{R}_{\mathrm{m}}) + (1 - R)\hat{\theta} d(\hat{R}_{\mathrm{d}}) + (R - \hat{R}_{\mathrm{d}})d(\hat{R}_{\mathrm{d}})$, with $\hat{\theta} \equiv \hat{\theta}_{\mathrm{m}} + \hat{\theta}_{\mathrm{b}}$, $\hat{R}_{\mathrm{m}} > \bar{R}_{\mathrm{m}}$, $\hat{R}_{\mathrm{d}} > \bar{R}_{\mathrm{d}}$, $1/\hat{p}(1) \geq 1/\bar{p}(1)$, and $\hat{G} \geq \bar{G}$.

Suppose we choose an alternative single reserve ratio $\tilde{\theta}$ such that $\tilde{R}_{\mathrm{d}} \equiv \tilde{\theta}\hat{R}_{\mathrm{m}} + (1 - \tilde{\theta})R = \hat{R}_{\mathrm{d}}$. The multiple-reserve deposit-rate equation implies that this choice requires $\tilde{\theta}\hat{R}_{\mathrm{m}} + (1 - \tilde{\theta})R = \hat{\theta}_{\mathrm{m}}\hat{R}_{\mathrm{m}} + \hat{\theta}_{\mathrm{b}}\hat{R}_{\mathrm{b}} + (1 - \hat{\theta}_{\mathrm{m}} - \hat{\theta}_{\mathrm{b}})R \Leftrightarrow \tilde{\theta}(\hat{R}_{\mathrm{m}} - R) = (\hat{\theta}_{\mathrm{m}} + \hat{\theta}_{\mathrm{b}})(\hat{R}_{\mathrm{m}} - R) + \hat{\theta}_{\mathrm{b}}(\hat{R}_{\mathrm{b}} - \hat{R}_{\mathrm{m}})$.

Case 1. $\hat{R}_{\mathrm{b}} > \hat{R}_{\mathrm{m}}$. In this case $\tilde{\theta}(\bar{R}_{\mathrm{m}} - R) = (\hat{\theta}_{\mathrm{m}} + \hat{\theta}_{\mathrm{b}})(\hat{R}_{\mathrm{m}} - R) + \hat{\theta}_{\mathrm{b}}(\hat{R}_{\mathrm{b}} - \hat{R}_{\mathrm{m}}) \Leftrightarrow (\tilde{\theta} - \hat{\theta})(\hat{R}_{\mathrm{m}} - R) > 0 \Leftrightarrow \tilde{\theta} < \hat{\theta}$. Since $\tilde{R}_{\mathrm{d}} = \hat{R}_{\mathrm{d}}$, the equilibrium government budget equations now imply that $\tilde{G} > \hat{G}$; since $\hat{G} \geq \bar{G}$ by assumption, they also imply that $\tilde{G} > \bar{G}$. In single-reserve requirement equilibria, moreover, we can write $G = (1 - R_{\mathrm{m}})A \Leftrightarrow A = G/(1 - R_{\mathrm{m}}) \Leftrightarrow$ (using the money demand

[20] See Bullard and Russell (1999).
[21] For estimates of the amount of U.S. government currency circulating abroad, see Porter and Judson (1996).

equation) $M(0)/p(1) = R_{\mathrm{m}}G/(1 - R_{\mathrm{m}})$. Thus $\tilde{G} > \bar{G}$ and $\tilde{R}_{\mathrm{m}} > \bar{R}_{\mathrm{m}}$ implies that $1/\tilde{p}(1) > 1/\bar{p}(1)$.

Case 2. $\hat{R}_{\mathrm{b}} < \hat{R}_{\mathrm{m}} = \bar{R}_{\mathrm{m}}$. In this case $\tilde{\theta}(\bar{R}_{\mathrm{m}} - R) = (\hat{\theta}_{\mathrm{m}} + \hat{\theta}_{\mathrm{b}})(\hat{R}_{\mathrm{m}} - R) + \hat{\theta}_{\mathrm{b}}(\hat{R}_{\mathrm{b}} - \bar{R}_{\mathrm{m}}) \Leftrightarrow \tilde{\theta} > \hat{\theta}$. Since $\tilde{R}_{\mathrm{d}} = \hat{R}_{\mathrm{d}}$, the money demand and government budget equations imply that $M(0)/\tilde{p}(1) = \tilde{A} - \tilde{G} = \bar{R}_{\mathrm{m}}[m(\bar{R}_{\mathrm{m}}) + \tilde{\theta}d(\hat{R}_d)]$, and $M(0)/\hat{p}(1) = \hat{A} - \hat{G} = \bar{R}_{\mathrm{m}}[m(\bar{R}_m) + \hat{\theta}_{\mathrm{m}}d(\hat{R}_{\mathrm{d}})] + \hat{R}_{\mathrm{b}}\hat{\theta}_{\mathrm{b}}d(\hat{R}_{\mathrm{d}})$. Thus $M(0)/\tilde{p}(1) - M(0)/\hat{p}(1) = [\bar{R}_{\mathrm{m}}\tilde{\theta} - (\bar{R}_{\mathrm{m}}\hat{\theta}_{\mathrm{m}} + \hat{R}_{\mathrm{b}}\hat{\theta}_{\mathrm{b}})]d(\hat{R}_{\mathrm{d}})$, so $\bar{R}_{\mathrm{m}} > \hat{R}_{\mathrm{b}}$ and $\tilde{\theta} > \hat{\theta}$ imply that $1/\tilde{p}(1) > 1/\hat{p}(1)$, and since $1/\hat{p}(1) \geq 1/\bar{p}(1)$ by assumption, we have $1/\tilde{p}(1) > 1/\bar{p}(1)$. As we have just seen, for a single-reserve requirement $M(0)/p_1 = R_{\mathrm{m}}G/(1 - R_{\mathrm{m}})$. Thus, $1/\tilde{p}(1) > 1/\bar{p}(1)$ and $\tilde{R}_m = \bar{R}_m \Rightarrow \tilde{G} > \bar{G}$. ■

Proof of Proposition 2. $1/\hat{p}_1 = 1/\bar{p}_1$ requires that $\bar{A} - \bar{G} = \hat{A} - \hat{G}$, and $\bar{G} = \hat{G}$ then implies that $\bar{A} = \hat{A}$. This equality, together with $\hat{R}_{\mathrm{d}} = \bar{R}_{\mathrm{d}}$ and $\hat{R}_{\mathrm{m}} = \bar{R}_{\mathrm{m}}$, implies that $\hat{\theta} = \bar{\theta}$. These three equalities imply that $\bar{R}_{\mathrm{m}}\hat{\theta}_{\mathrm{b}} + \hat{R}_{\mathrm{b}}\hat{\theta}_{\mathrm{b}} = \bar{R}_{\mathrm{m}}\bar{\theta}_{\mathrm{m}} + \bar{R}_{\mathrm{b}}\bar{\theta}_{\mathrm{b}}$, and this equation can be used, along with $\hat{\theta} = \bar{\theta}$, to produce

$$\hat{\theta}_{\mathrm{b}} = \frac{\bar{R}_{\mathrm{b}} - \bar{R}_{\mathrm{m}}}{\hat{R}_{\mathrm{b}} - \bar{R}_{\mathrm{m}}}\,\bar{\theta}_{\mathrm{b}}.$$

Case 1. Suppose $\bar{R}_{\mathrm{b}} < \bar{R}_{\mathrm{m}}$. Since $\hat{\theta}_{\mathrm{b}}$ must be nonnegative we must have $\hat{R}_{\mathrm{b}} < \bar{R}_{\mathrm{m}} = \hat{R}_{\mathrm{m}}$, which is an unconventional regime. The minimum value for $\hat{\theta}_{\mathrm{b}}$ consistent with $\hat{\theta}_{\mathrm{b}} \in [0, \bar{\theta}]$ occurs when $\hat{R}_{\mathrm{b}} = 0$, which produces $\hat{\theta}_{\mathrm{b}}^{\min} = \bar{\theta}_{\mathrm{b}}(1 - \bar{R}_{\mathrm{b}}/\bar{R}_{\mathrm{m}})$. Since $\hat{\theta}_{\mathrm{b}}^{\min} > 0$ there is always an equilibrium of this type. The maximum value for $\hat{\theta}_{\mathrm{b}}$ is $\bar{\theta}$, which is produced by $\hat{R}_{\mathrm{b}}^{\max} = (\bar{R}_{\mathrm{m}}\bar{\theta}_{\mathrm{m}} + \bar{R}_{\mathrm{b}}\bar{\theta}_{\mathrm{b}})/\bar{\theta}$. It is readily seen that $\bar{R}_{\mathrm{b}} \leq \hat{R}_{\mathrm{b}}^{\max} < \bar{R}_{\mathrm{m}}$, so there is always an equilibrium of this type. It follows that there is an equilibrium for every $\hat{R}_{\mathrm{b}} \in [0, \hat{R}_{\mathrm{b}}^{\max}]$.

Case 2. Suppose $\bar{R}_{\mathrm{b}} > \bar{R}_{\mathrm{m}}$. In this case, $\hat{\theta}_{\mathrm{b}}$ nonnegative requires $\hat{R}_{\mathrm{b}} > \bar{R}_{\mathrm{m}} = \hat{R}_{\mathrm{m}}$, which is a conventional regime. The minimum value for $\hat{\theta}_{\mathrm{b}}$ consistent with equilibrium is produced by $\hat{R}_{\mathrm{b}} = R$; it is $\hat{\theta}_{\mathrm{b}}^{\min} = (\bar{R}_{\mathrm{b}} - \bar{R}_{\mathrm{m}})/(R - \bar{R}_{\mathrm{m}})\bar{\theta}_{\mathrm{b}}$. Since $\hat{\theta}_{\mathrm{b}}^{\min} \in (0, \bar{\theta}_{\mathrm{b}}]$ there is always an equilibrium of this type. The maximum value for $\hat{\theta}_{\mathrm{b}}$ consistent with equilibrium is $\bar{\theta}$, which is produced by $\hat{R}_{\mathrm{b}}^{\min} = (\bar{R}_{\mathrm{m}}\bar{\theta}_{\mathrm{m}} + \bar{R}_{\mathrm{b}}\bar{\theta}_{\mathrm{b}})/\bar{\theta}$. In this case we have $\bar{R}_{\mathrm{m}} < \hat{R}_{\mathrm{b}}^{\min} \leq \bar{R}_{\mathrm{b}}$, so there is always an equilibrium of this type. It follows that there is an equilibrium for every $\hat{R}_{\mathrm{b}} \in [\hat{R}_{\mathrm{b}}^{\min}, R]$. ■

Examples

For the purposes of these examples, define the poor savers' seigniorage revenue function (Laffer curve) as $G_{\mathrm{p}}(R_{\mathrm{m}}) \equiv (1 - R_{\mathrm{m}})m(R_{\mathrm{m}})$. The rich savers' version of this function is $G_{\mathrm{r}}(R_{\mathrm{m}}, \theta) \equiv (1 - R_{\mathrm{m}})\,\theta d(R_{\mathrm{d}}(R_{\mathrm{m}}, \theta))$, where $R_{\mathrm{d}}(R_{\mathrm{m}}, \theta) \equiv (1 - \theta)R + \theta R_{\mathrm{m}}$. The aggregate seigniorage revenue function is $G_{\mathrm{r}}(R_{\mathrm{m}}, \theta) \equiv G_{\mathrm{p}}(R_{\mathrm{m}}) + G_{\mathrm{r}}(R_{\mathrm{m}}, \theta)$. In the latter two cases, holding θ fixed and varying R_{m} produces a "currency-return-rate Laffer curve," while holding R_{m} fixed and varying θ produces a "reserve-ratio Laffer curve." To avoid confusion, we will often refer to the poor savers' seigniorage revenue function as their currency-return-rate Laffer curve.

If a value of R_{m} is on the "wrong" (left-hand) side of a currency-return-rate Laffer curve, then increasing R_{m} while holding θ fixed will increase seigniorage revenue. If a value of θ is on the "wrong" (right-hand) side of a reserve-ratio Laffer curve, then decreasing θ while holding R_{m} fixed will increase seigniorage revenue.

The (indirect) social utility functions we use in these examples have the form $W(m_0, R_{\mathrm{d}}, R_{\mathrm{m}}) = a_0 \log(m_0/\bar{m}) + a_1 \log(R_{\mathrm{d}}/R) + a_2 \log R_{\mathrm{m}}$, where $m_0 = M(0)/p(1)$, $\bar{m} = m(1) + d(1) - G$, $a_i > 0$ for $i = 0, 1, 2$, and $\sum_{i=1}^{3} a_i = 1$.

EXAMPLE 1. Let $R = 1.2$, $m(R_{\mathrm{m}}) = 5(5 - 2/R_{\mathrm{m}})$, $d(R_{\mathrm{d}}) = 16\,(48 - 43/R_{\mathrm{d}})$, $M(0) = 1$, and $G = 10$. Let the parameters of the social utility function be $a_0 = 0.2$, $a_1 = 0.75$, and $a_2 = 0.05$. The social-utility-maximizing single-reserve allocation includes $R_{\mathrm{m}} \doteq 0.8250$, $R_{\mathrm{d}} \doteq 1.062$, and $m_0 \doteq 47.13$, producing social utility $W \doteq 0.7805$. The single-reserve policy setting that supports this allocation features an aggregate reserve ratio of $\theta \doteq 0.3689$. The social-utility-maximizing multiple-reserve allocation features $R_{\mathrm{m}} \doteq 0.7440$, $R_{\mathrm{d}} \doteq 1.061$, and $m_0 \doteq 49.41$, producing social utility $W \doteq 0.7845$. The multiple-reserve policy setting that supports this allocation features an aggregate reserve ratio $\theta \doteq 0.3998$. The values $(\theta_{\mathrm{m}}, R_{\mathrm{b}})$ that support this allocation are not unique: they lie along a line segment in $\mathbf{R}^2$ connecting (0, 0.8529) to (0.304318, 1.2).

EXAMPLE 2. Let $R = 1.2$, $m(R_{\mathrm{m}}) = 5\,(5 - 2/R_{\mathrm{m}})$, $d(R_{\mathrm{d}}) = 16(48 - 43/R_{\mathrm{d}})$, $M(0) = 1$, and $G = 14.75$. Let the parameters of the social utility function be $a_0 = 0.1999$, $a_1 = 0.8$, and $a_2 = 0.0001$. The single-reserve allocation that maximizes social utility includes $\bar{R}_{\mathrm{m}} = 0.5886$, $\bar{R}_{\mathrm{d}} \doteq 1.0418$, and $\bar{m}_0 \doteq 21.11$. The policy setting that supports this allocation features $\bar{\theta} \doteq 0.2588$. The equilibrium value of R_{m} is on the "wrong" (left-hand) side of the currency return rate Laffer curve for poor savers, which peaks at $R_{\mathrm{m}} \doteq 0.6325$. It is readily seen, however, that the equilibrium value of R_{m} is on the "right" side of the aggregate currency-return-rate Laffer curve, so that increases in R_{m} reduce total seigniorage revenue if θ is held constant. The initial equilibrium is also on the "right" (right-hand) side of the reserve-ratio Laffer curve, so that reducing θ while holding R_{m} fixed will not produce any additional revenue, and while it is possible to increase θ by an amount that restores the revenue lost by a small increase in R_{m}, any such increase produces a decline in R_{d} and hurts the rich savers. For example, the largest value of R_{m} that will support a single-reserve equilibrium is approximately 0.5909. (Note that this value remains on the wrong side of the poor savers' currency-return-rate Laffer curve.) When R_{m} is set at this value, the value of θ needed to finance G is approximately $0.2677 > \bar{\theta}$, which produces $R_{\mathrm{d}} \doteq 1.0370 < \bar{R}_{\mathrm{d}}$. Thus, the initial single-reserve allocation is efficient relative to all other single-reserve allocations.

If the government switches to a multiple-reserve regime, then it can increase R_{m} to 0.6325, the value corresponding to the peak of the poor savers' currency-return-rate Laffer curve, without losing any seigniorage revenue from rich savers. For this value of R_{m}, the optimal multiple-reserve regime features $\theta \doteq 0.2490$, and the associated allocation includes $R_d \doteq 1.0485$ and $m_0 \doteq 22.28$. Thus, this allocation Pareto dominates the optimal single-reserve allocation.

The optimal multiple-reserve policy setting in this economy features $R_m \doteq 0.6929$ and $\theta \doteq 0.2423$. The associated allocation includes $R_d \doteq 1.0489$ and $m_0 \doteq 22.97$, so it Pareto dominates both the optimal single-reserve allocation and the optimal multiple-reserve allocation when R_m is set at the value associated with the peak of the poor savers' Laffer curve. The policy settings (θ_m, R_b) that support this allocation lie along a line segment connecting (0, 0.5763) to (0.2016, 0).

EXAMPLE 3. Let $R = 1.2$, $m(R_m) = 2(5 - 2/R_m)$, $d(R_d) = 48 - 43/R_d$, $M(0) = 1$, and $G = 1.9233$. This economy has a single-reserve equilibrium that was described by Espinosa (1995): it involves setting R_m at 0.65 and θ_m at 0.4, and it produces $R_d = 0.98$ and $m_0 \doteq 3.572$. However, the allocation supported by this equilibrium is Pareto-dominated by the equilibrium allocation associated with a single-reserve policy setting that keeps R_m at 0.65 but reduces θ_m to 0.1874. This equilibrium keeps m_0 unchanged but allows R_d to increase to 1.097.

In his example, Espinosa (1995) showed that the allocation produced by a multiple-reserve policy setting with $R_b = 1.1\, R_m$, $\theta_m = 0.3$, and $\theta_b = 0.1$ also Pareto dominates his single-reserve allocation. However, Espinosa's multiple-reserve allocation does *not* Pareto dominate the allocation supported by the alternative single-reserve equilibrium just described. Espinosa's equilibrium values of R_m and m_0 are approximately 0.6748 and 4.091, respectively, but his equilibrium value of R_d is only 0.9967.

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